Going into AP Calculus AB, there are certain skills that have been taught to you over the previous years that I assume you have. If you do not have these skills, you will find that you will consistently get problems incorrect next year, even though you understand the calculus concepts. It is frustrating for students when they are tripped up by the algebra and not the calculus. This summer packet is intended for you to brush up and possibly relearn these topics.

I assume that you have basic skills in algebra. Being able to solve equations, work with algebraic expressions, and basic factoring, for example, should now be a part of you. If not, you would not be going into AP Calculus. So, only the topics I see that students consistently do not have down in their basic skills set are included here. These are skills that are used continually in AP Calculus.

On the following 16 pages, you have 6 to 12 problems per page. Each problem should be done in the space provided. Rather than give you a textbook to remind you of the techniques necessary to solve the problem, I have given you several websites that have full instructions on the techniques. If and when you are unsure of how to attempt these problems, examine these websites. Don't fake your way through these problems. As stated, students are notoriously weak in these topics, even students who have achieved well prior to AP Calculus. Use the websites!

Realize also that certain concepts are interrelated. Domain, for example, may require you to be an expert at working with inequalities. Solving quadratic equations may involve techniques used in solving fractional equations.

This packet is due the first day back to school in the fall. It will be graded for completion. You need to get off to a good start, so spend some quality time on this packet this summer. Be sure your name appears on the first sheet. Work needs to be shown when needed and be neat. Also, do not rely on your calculator. Two thirds of the AP exam is taken without the calculator. Paper and pencil techniques are the only option on these problems.

It is a mistake to the decide to do this at the beginning of the summer. Let it go to mid-summer. I want these techniques to be relatively fresh in your mind in the fall. Also, do not wait to do them at the very last minute. These take time. If you do two concepts a day, the whole packet will take you about a week to complete.

If you have any questions about any of these problems or techniques used in solving them, contact me at: bhellickson@c-dh.org. Have a great summer and I will see you next fall!

Mrs Hellickson

The topics are listed below. You can certainly do Google searches for any of these topics. However, I have provided several websites that will cover pretty much all of these topics.

Websites:
https://www.khanacademy.org/

https://www.purplemath.com/index.htm

Beginning Algebra Topics:
1. Exponents - Negative and Fractional Exponents

Intermediate Algebra Topics:
2. Domain
3. Solving Inequalities – Absolute Value
4. Solving Inequalities – Quadratic
5. Special Factoring Formulas
6. Function Transformations
7. Rational Root Theorem (p/q method)
8. Even/Odd Functions
9. Solving Quadratic Equations

Advanced Algebra Topics:
10. Asymptotes
11. Complex Fractions
12. Composition of Functions
13. Solving Rational Equations

Trigonometry:
1. Basic Right Triangle Trigonometry
2. Solving Trigonometric Equations
3. Graphs of all 6 Trig Functions
4. Exact Values of Trig Functions for $\frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6}$ and axis angles

Topic #1: Fractional and Negative Exponents

Simplify using only positive exponents.

1.) $-3x^{-3}$

2.) $-5\left(\frac{3}{2}\right)(4-9x)^{-1/2} \cdot (-9)$

3.) $2\left(\frac{2}{2-x}\right)\left(\frac{-2}{(2-x)^2}\right)$

4.) $(16x^2y)^{3/4}$

5.) $-\frac{x^{-1/2}}{2}\sin\sqrt{x}$

6.) $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$

7.) $-4\left(\frac{2x-1}{2x+1}\right)^{-3}\left[\frac{2(2x+1)-2(2x-1)}{(2x+1)^2}\right]$

8.) $\left(\frac{1}{x^{-2}}+\frac{4}{x^{-1}y^{-1}}+\frac{1}{y^{-2}}\right)^{-1/2}$

9.) $\frac{\frac{1}{2}(2x+5)^{-3/2}}{\frac{3}{2}}$

Topic 2: Domain

Find the domain of the following functions. Write domain in interval notation.

1.) $y = \dfrac{3x-2}{4x+1}$

2.) $y = \dfrac{x^2-4}{2x+4}$

3.) $y = \dfrac{x^2-5x-6}{x^2-3x-18}$

4.) $y = \dfrac{2^{2-x}}{x}$

5.) $y = \sqrt{x-3} - \sqrt{x+3}$

6.) $y = \dfrac{\sqrt{2x-9}}{2x+9}$

7.) $y = \dfrac{x^2+8x+12}{\sqrt[4]{x+5}}$

8.) $y = \log(2x-12)$

9.) $y = \sqrt{\tan x}$

10.) $y = \dfrac{x}{\cos x}$

Topics 3: Absolute Values

Solve the following absolute value inequalities. Answer using interval notation.

1.) $|x - 3| > 12$

2.) $|x - 3| \leq 4$

3.) $|10x + 8| > 2$

4.) $|3x - 4| > -2$

5.) $|x - 6| \leq -8$

6.) $|x + 1| \leq |x - 3|$

Topic 4: Solving Quadratic Inequalites

Solve the following inequalities by factoring and making the appropriate sign line analysis. Answer in interval notation.

1.) $x^2 - 16 > 0$ 2.) $x^2 + 6x - 16 > 0$

3.) $x^2 - 3x \geq 10$ 4.) $2x^2 + 4x \leq 3$

5.) $x^3 + 4x^2 - x \geq 4$ 6.) $2 \sin^2 x \geq \sin x$ $0 \leq x < 2\pi$

Topic 5: Special Factorization

Factor each expression completely.

1.) $x^3 + 8$ 2.) $x^3 - 8$ 3.) $27x^3 - 125y^3$

4.) $x^4 + 11x^2 - 80$ 5.) $ac + cd - ab - bd$ 6.) $2x^2 + 50y^2 - 20xy$

7.) $x^2 + 12x + 36 - 9y^2$ 8.) $x^3 - xy^2 + x^2y - y^3$

9.) $(x - 3)^2(2x + 1)^3 + (x - 3)^3(2x + 1)^2$ 10.) $\frac{1}{2}x^{-1/2}(3x + 4)^{1/2} + \frac{3}{2}x^{1/2}(3x + 4)^{-1/2}$

Topic 6: Function Transformation

If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of $f(x)$:

1.) $f(x) - 4$

2.) $f(x - 4)$

3.) $-f(x + 2)$

4.) $5f(x) + 3$

5.) $f(2x)$

6.) $|f(x)|$

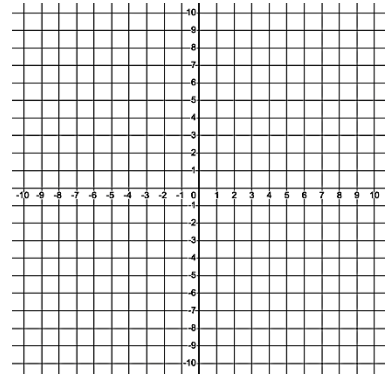To the right is the graph of $y = f(x)$. Sketch the following graphs.
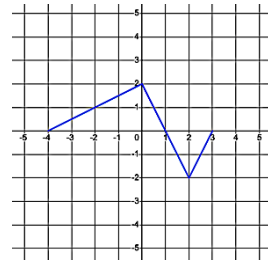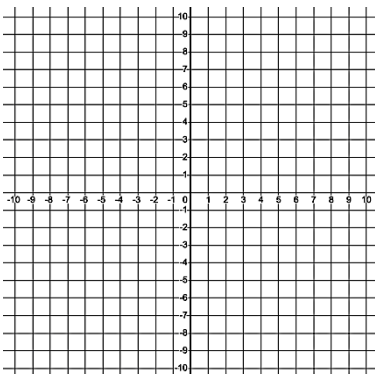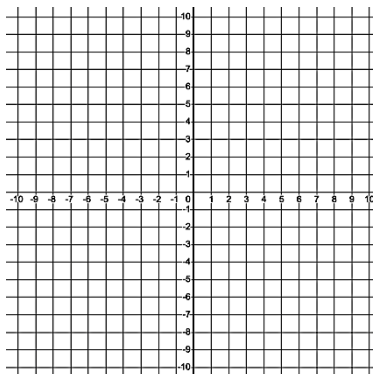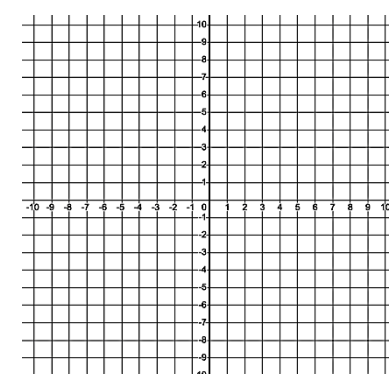
7.) $y = 2f(x)$
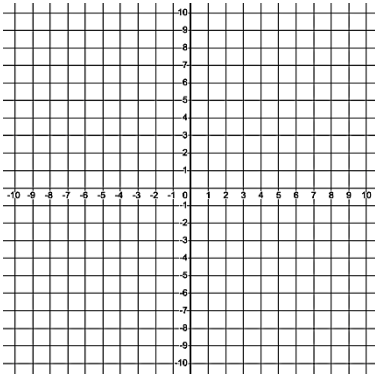
8.) $y = -f(x)$
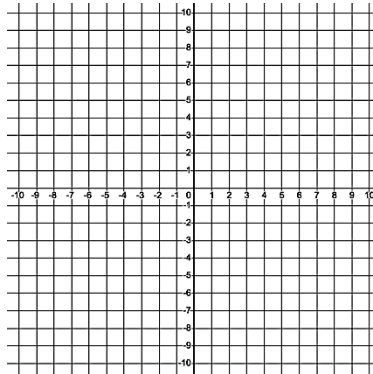
9.) $y = f(x - 1)$

10.) $y = f(x + 2)$

11.) $y = |f(x)|$

12.) $y = f|x|$

Topic 7: Rational Root Theorem (p/q method and Synthetic Division)

Use the rational root theorem and synthetic division to factor the polynomial $P(x)$. Then solve $P(x) = 0$

1.) $P(x) = x^3 + 4x^2 + x - 6$

2.) $P(x) = x^3 + 5x^2 - 2x - 24$

3.) $P(x) = x^3 - 6x^2 + 3x - 18$

4.) $P(x) = x^3 + 2x^2 - 19x - 20$

5.) $P(x) = x^4 + 5x^3 + 6x^2 - 4x - 8$

6.) $P(x) = x^4 + 11x^3 + 41x^2 + 61x + 30$

Topic 8: Even and Odd Functions

Show work to determine if the relation is even, odd, or neither.

1.) $f(x) = 2x^2 - 7$

2.) $f(x) = -4x^3 - 2x$

3.) $f(x) = 4x^2 - 4x + 4$

4.) $f(x) = x - \frac{1}{x}$

5.) $f(x) = |x| - x^2 + 1$

6.) $5x^2 - 6y = 1$

7.) $y = e^x - \frac{1}{e^x}$

8.) $3y^3 = 4x^3 + 1$

9.) $3x = |y|$

Topic 9: Solving Quadratic Equations and Using Quadratic Formula

Solve each equation.

1.) $7x^3 - 3x = 0$

2.) $4x(x - 2) - 5x(x - 1) = 2$

3.) $x^2 + 6x + 4 = 0$

4.) $2x^2 - 3x + 3 = 0$

5.) $2x^2 - (x + 2)(x - 3) = 12$

6.) $x + \frac{1}{x} = \frac{13}{6}$

7.) $x^4 - 9x^2 + 8 = 0$

8.) $x - 10\sqrt{x} + 9 = 0$

9.) $\frac{1}{x^2} - \frac{1}{x} = 6$

Topic 10: Asymptotes

For each function, find the equations of both the vertical asymptote(s) and the horizontal asymptote(s), if they exist.

1.) $y = \dfrac{x}{x-3}$

2.) $y = \dfrac{x+4}{x^2-1}$

3.) $y = \dfrac{x+4}{x^2+1}$

4.) $y = \dfrac{x^2-2x+1}{x^2-3x-4}$

5.) $y = \dfrac{x^2-9}{x^3+3x^2-18x}$

6.) $y = \dfrac{2x^2+6x}{x^3-3x^2-4x}$

7.) $y = \dfrac{x^2-x-6}{x^3-x^2+x-6}$

8.) $y = \dfrac{2x^3}{x^3-1}$

9.) $y = \dfrac{\sqrt{x}}{2x^2-10}$

Topic 11: Complex Fractions

Simplify the following.

1.) $\dfrac{x}{x-\frac{1}{2}}$

2.) $\dfrac{\frac{1}{x}+4}{\frac{1}{x}-2}$

3.) $\dfrac{x-\frac{1}{x}}{x+\frac{1}{x}}$

4.) $\dfrac{\frac{3}{x}-\frac{4}{y}}{\frac{4}{x}-\frac{3}{y}}$

5.) $\dfrac{1-\frac{2}{3x}}{x-\frac{4}{9x}}$

6.) $\dfrac{\frac{x^2-y^2}{xy}}{\frac{x+y}{y}}$

7.) $\dfrac{x^{-3}-x}{x^{-2}-1}$

8.) $\dfrac{\frac{x}{1-x}+\frac{1+x}{x}}{\frac{1-x}{x}+\frac{x}{1+x}}$

9.) $\dfrac{\frac{4}{x-5}+\frac{2}{x+2}}{\frac{2x}{x^2-3x-10}+3}$

Topic 12: Composition of Functions

If $f(x) = x^2$, $g(x) = 2x - 1$, and $h(x) = 2^x$, find each of the following.

1.) $f(g(2))$

2.) $g(f(2))$

3.) $f(h(-1))$

4.) $h(f(-1))$

5.) $g\left(f\left(h\left(\frac{1}{2}\right)\right)\right)$

6.) $f(g(x))$

7.) $g(f(x))$

8.) $g(g(x))$

9.) $f(h(x))$

Topic 13: Solving Rational Equations

Solve each equation for x (without using a calculator)

1.) $\frac{2}{3} - \frac{5}{6} = \frac{1}{x}$

2.) $x + \frac{6}{x} = 5$

3.) $\frac{x+1}{3} - \frac{x-1}{2} = 1$

4.) $\frac{x-5}{x+1} = \frac{3}{5}$

5.) $\frac{60}{x} - \frac{60}{x-5} = \frac{2}{x}$

6.) $\frac{2}{x+5} + \frac{1}{x-5} = \frac{16}{x^2-25}$

7.) $\frac{x}{x-2} + \frac{2x}{4-x^2} = \frac{5}{x+2}$

8.) $\frac{x}{2x-6} - \frac{3}{x^2-6x+9} = \frac{x-2}{3x-9}$

9.) $\frac{2x+3}{x-1} = \frac{10}{x^2-1} + \frac{2x-3}{x+1}$

Topic 14: Solving Trig Problems

Solve the following.

If point P is on the terminal side of θ in standard position, find all 6 trig functions of θ. Draw a sketc to help.

1.) $P(-2, 4)$

2.) $P(\sqrt{5}, -2)$

3.) If $\cos\theta = -\dfrac{5}{13}$ and θ is in quadrant II,
     Find $\sin\theta$ and $\tan\theta$

4.) If $\cot\theta = 3$ and θ is in quadrant III
     Find $\sin\theta$ and $\cos\theta$

Find the exact value of the following, without calculators.

5.) $\sin^2 225° - \cos^2 300°$

6.) $(6\sec 180° - 4\cot 90°)^2$

7.) $(4\cos 30° - 6\sin 120°)^{-2}$

Solve the following triangles (3 decimal place accuracy)

8.)
$\begin{aligned} A &= \qquad & a &= 21.7 \\ B &= 16° & b &= \\ C &= 90° & c &= \end{aligned}$

9.)
$\begin{aligned} A &= \qquad & a &= 6\ feet \\ B &= & b &= \\ C &= 90° & c &= 95\ feet \end{aligned}$

Topic 15: Solving Trigonometric Equations

Solve each of the following on the interval $[0, 2\pi)$. Give exact radian answers whenever possible; otherwise, round to the nearest hundredth.

1.) $\sin x = \dfrac{1}{2}$

2.) $\cos^2 x = \cos x$

3.) $2 \cos x + \sqrt{3} = 0$

4.) $4 \sin^2 x = 1$

5.) $2 \sin^2 x + \sin x = 1$

6.) $\cos^2 x + 2 \cos x = 3$

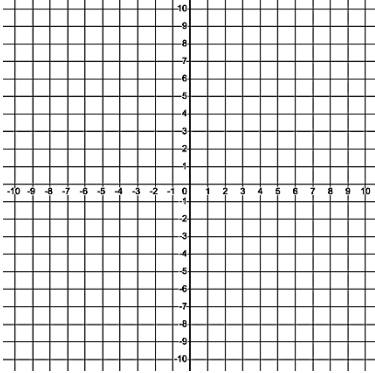7.) $2 \sin x \cos x + \sin x = 0$
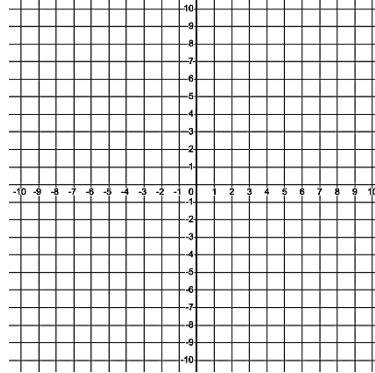
8.) $8 \cos^2 x - 2 \cos x = 1$

9.) $\sin^2 x - \cos^2 x = 0$

Topic 16: Important Graphs

Graph each of the following. Identify any asymptotes you find.
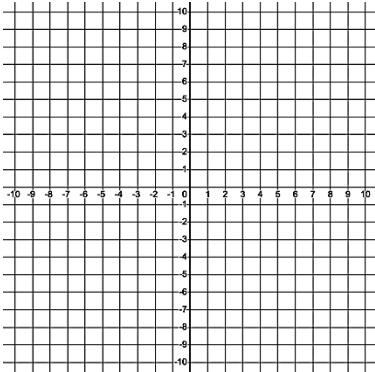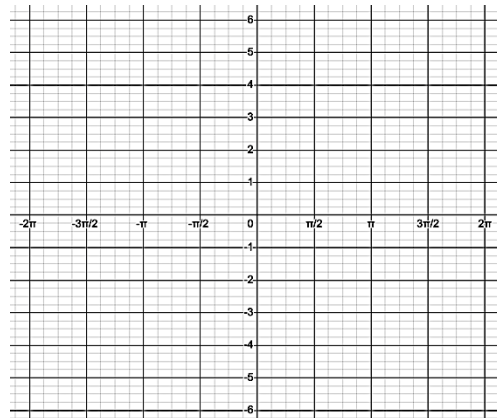
1.) $y = e^x$
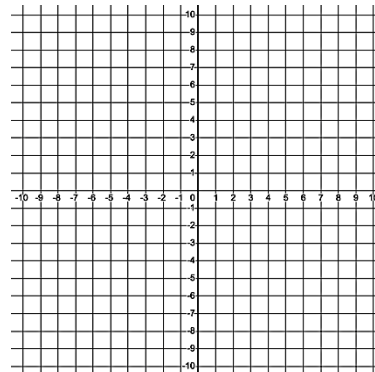
2.) $y = e^{-x}$

3.) $y = \ln x$

4.) $y = \tan x$

5.) $f(x) = \begin{cases} x + 2, & x \leq -1 \\ x^2, & x > -1 \end{cases}$

6.) $y = |x|$