NOTES FOR

AP PHYSICS C

SUMMER 2016

CH. 1 MEASUREMENT

BASE QUANTITIES & SI UNITS:

	QUANTITY	UNIT NAME	UNIT ABBREVIATION
FUNDAMENTAL QUANTITIES	LENGTH	METER	m
	MASS	KILOGRAM	Kg
	TIME	SECOND	s

NOTE PREFIXES FOR SI UNITS ON P. 3.

WE WILL USE THESE OFTEN.

BE ABLE TO CONVERT BETWEEN DIFFERENT METRIC PREFIXES. (RARELY WE WILL CONVERT BETWEEN SI & ENGLISH SYSTEMS.)

FUNDAMENTAL VS. DERIVED QUANTITIES

FUNDAMENTAL QUANTITIES ARE THE MOST BASIC.
DERIVED QUANTITIES COME FROM FUNDAMENTAL.

EXAMPLE

$$\text{DENSITY} = \frac{\text{MASS}}{\text{VOLUME}}$$

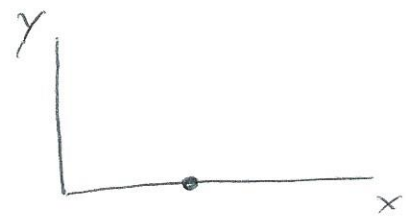
MASS, M IS FUNDAMENTAL
VOLUME IS DERIVED. IT IS 3 LENGTHS MULTIPLIED BY EACH OTHER ($L \times W \times h$)

SO, THE QUANTITY OF DENSITY IS DERIVED.

Ch. 2 Motion in 1 Dimension (1-D)

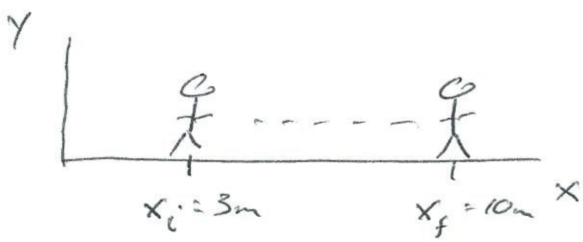
WE WILL USE THE X-Y COORDINATE SYSTEM EXTENSIVELY IN THIS COURSE.

POSITION OF AN OBJECT ~ X



DISPLACEMENT ~ CHANGE IN POSITION, ΔX

$$\Delta X = \underset{\text{(FINAL)}}{X_f} - \underset{\text{(INITIAL)}}{X_i} \quad \text{OR} \quad \Delta X = X_2 - X_1$$



$\Delta X = 7m$

DISTANCE - HOW FAR AN OBJECT TRAVELS, d

DISPLACEMENT & DISTANCE ARE NOT ALWAYS THE SAME!

EXAMPLE

IN BASEBALL, A BATTER HITS A HOME RUN. WHILE CIRCLING THE BASES, THE DISTANCE HE TRAVELS IS $d = 4 \times 90\,ft = 360\,ft$. HIS DISPLACEMENT, $\Delta X = 0$, SINCE $X_i = X_f$ (HE ENDS UP AT THE SAME POINT WHERE HE STARTED.)

SAME EXAMPLE COULD BE USED FOR SOMEONE WHO RUNS THE 400m IN TRACK. $d = 400m$ $\Delta X = 0$

SPEED & VELOCITY

NOT THE SAME THING, BUT CLOSELY RELATED.
BOTH GIVEN BY THE LETTER V.

SPEED, $V = \dfrac{\text{DISTANCE}}{\text{TIME INTERVAL}}$

TIME INTERVAL, Δt

$\Delta t = t_2 - t_1$

VELOCITY, $V = \dfrac{\text{DISPLACEMENT}}{\text{TIME INTERVAL}}$

$$\boxed{V = \frac{\Delta x}{\Delta t}}$$

UNITS: $\dfrac{m}{s}$

$$V = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

OFTEN, $t_1 = 0$ (WE BEGIN TIMING WHEN THE STOPWATCH IS AT 0.)

WE THEN DROP THE SUBSCRIPT ON t_2 SINCE WE HAVE ONLY ONE t.

$$V = \frac{x_2 - x_1}{t}$$

ALSO, SINCE x_1 IS THE POSITION AT t_1, & SINCE $t_1 = 0$, WE THEN CHANGE x_1 TO x_0.

x_0 IS PRONOUNCED "X-NAUGHT".

x_0 IS THE POSITION WHEN $t = 0$.

NOW, x_2 IS THE POSITION AT TIME t. SINCE t HAS NO SUBSCRIPT, WE DROP THE SUBSCIPT ON x_2.

$$\boxed{V = \frac{x - x_0}{t}}$$

$$V = \frac{X - X_0}{t}$$

~ THIS RELATIONSHIP IS GOOD FOR AN OBJECT MOVING AT CONSTANT VELOCITY, THAT IS, NO ACCELERATION.

TO FIND IT'S POSITION AT TIME, t:

$$\boxed{X = Vt + X_0}$$

X_0 ~ INITIAL POSITION

X ~ POSITION AT TIME, t

ACCELERATION, a, IS A CHANGE IN VELOCITY OVER SOME TIME INTERVAL.

$$\boxed{a = \frac{\Delta V}{\Delta t}}$$

UNITS: $\frac{m/s}{s} = m/s^2$

$$a = \frac{\Delta V}{\Delta t} = \frac{V_2 - V_1}{t_2 - t_1}$$

USING SAME IDEAS ($t_1 = 0, \ldots$) AS ON P. 3 OF NOTES:

$$\boxed{a = \frac{V - V_0}{t}}$$

TO FIND VELOCITY AT TIME, t:

$$\boxed{V = at + V_0}$$

V_0 ~ INITIAL VELOCITY

V ~ VELOCITY AT TIME, t

NOTICE: VELOCITY IS CHANGING. ON THE TOP OF THIS PAGE & BEFORE, VELOCITY IS CONSTANT.

WHEN VELOCITY IS CHANGING, WE HAVE ACCELERATION. IF IT CHANGES AT A CONSTANT RATE, WE HAVE CONSTANT ACCELERATION. WE WILL DEAL WITH MOSTLY A CONSTANT ACCELERATION IN THIS CLASS.

FOR A CONSTANT a, WE HAVE V_0 & V.

AVERAGE VELOCITY, $\boxed{\bar{V} = \frac{V_0 + V}{2}}$

DERIVATION OF KINEMATIC RELATIONSHIPS

WE WILL NOW DERIVE OTHER FORMULAS BASED ON THE ONES NOTED ABOVE. THESE WILL BE USEFUL TO SOLVE MOST PROBLEMS FOR 1-D MOTION.

GIVEN

$$x = Vt + x_0 \qquad \text{IF } V = \bar{V} = \frac{V_0 + V}{2}$$

$$x = \left(\frac{V_0 + V}{2}\right)t + x_0 \qquad V = at + V_0$$

$$x = \left(\frac{V_0 + at + V_0}{2}\right)t + x_0$$

$$\boxed{x = \tfrac{1}{2}at^2 + V_0 t + x_0}$$

THIS ALLOWS US TO FIND THE POSITION OF AN OBJECT IF IT IS ACCELERATING, AT SOME TIME t.

GIVEN

$$x = v t + x_0 \qquad v = \bar{v} = \frac{v_0 + v}{2}$$

$$x = \left(\frac{v_0 + v}{2}\right) t + x_0 \qquad v = at + v_0 \Rightarrow t = \frac{v - v_0}{a}$$

$$x = \left(\frac{v_0 + v}{2}\right)\left(\frac{v - v_0}{a}\right) + x_0$$

$$x = \frac{1}{2a}(v_0 + v)(v - v_0) + x_0$$

$$x = \frac{1}{2a}(v^2 - v_0^2) + x_0$$

$$-x_0 \qquad\qquad -x_0$$

$$\Delta x = \frac{1}{2a}(v^2 - v_0^2)$$

$$\boxed{v^2 = v_0^2 + 2a\Delta x}$$

THIS ALLOWS US TO FIND THE SPEED v OF AN OBJECT THAT ACCELERATES OVER SOME DISPLACEMENT, Δx.

EXAMPLES FOLLOW ON HOW TO USE THESE RELATIONSHIPS.

HELPFUL HINT: ALWAYS INDICATE WHAT YOU ARE GIVEN, WHAT YOU ARE TRYING TO FIND, & WHAT FORMULA(S) YOU NEED.

EXAMPLE 1 CAR GOING THROUGH AN INTERSECTION

HOW LONG DOES IT TAKE A CAR TO CROSS A 30 m WIDE INTERSECTION AFTER THE LIGHT TURNS GREEN, IF THE CAR ACCELERATES FROM REST AT A CONSTANT $a = 2\ m/s^2$?

GIVEN:

$v_0 = 0$

$\Delta x = 30m$

$a = 2\ m/s^2$

FIND:

t

FORMULA:

$$x = \frac{1}{2} a t^2 + v_0 t + x_0$$

SOLUTION:

SINCE $\Delta x = x - x_0 = 30m$, LET $x_0 = 0$. $x = 30$

$$x = \frac{1}{2} a t^2 + \cancel{v_0}^{0} t + \cancel{x_0}^{0}$$

$$x = \frac{1}{2} a t^2$$

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(30)}{2}}$$

$t = 5.5s$

EXAMPLE 2 AIR BAG DESIGN

HOW MUCH TIME DOES IT TAKE AN AIR BAG TO DEPLOY IF A CAR WERE GOING 28 m/s AND THE ACCELERATION OF THE AIR BAG IS 100 m/s²?

GIVEN:

$V_0 = 28\, m/s$

$V = 0$

$a = 100\, m/s^2$

FIND:

t

FORMULA:

$$V = at + V_0 \Rightarrow t = \frac{V - V_0}{a}$$

$$t = \frac{0 - 28}{-100}$$

a IS NEGATIVE BECAUSE IT IS DIRECTED IN THE OPPOSITE DIRECTION THE CAR IS MOVING.

$t = .28\, s$

EXAMPLE 3 BRAKING DISTANCE

SUPPOSE YOU ARE DRIVING IN A CAR AT $75 \frac{km}{hr}$ WHEN AN OBJECT APPEARS IN THE ROAD AHEAD OF YOU. IT TAKES 0.6 s FOR YOUR BRAIN TO REACT, AND THEN YOU ACCELERATE AT $-6\ m/s^2$ (DECELERATE) UNTIL YOU STOP. HOW FAR DO YOU GO IN THIS EVENT?

GIVEN:

THERE ARE TWO PARTS IN THIS PROBLEM - ① WHILE YOUR BRAIN REACTS & ② WHILE YOU HIT THE BRAKES

$V = 75 \frac{km}{hr} = 20.8\ m/s$

FIND:

$\Delta X_{TOTAL} = \Delta X_{1} + \Delta X_{2}$

FOR PART ① - CONSTANT VELOCITY

$$V = \frac{\Delta X_1}{t} \qquad \Delta X_1 = Vt = 20.8(.6)$$

$$\Delta X_1 = \underline{12.5\,m}$$

FOR PART ②

$V_0 = 20.8\ m/s$

$V = 0$

$a = -6\ m/s^2$

FIND ΔX_2

FORMULA:

$$V^2 = V_0^2 + 2a\Delta X$$

$$\Delta X = \frac{V^2 - V_0^2}{2a}$$

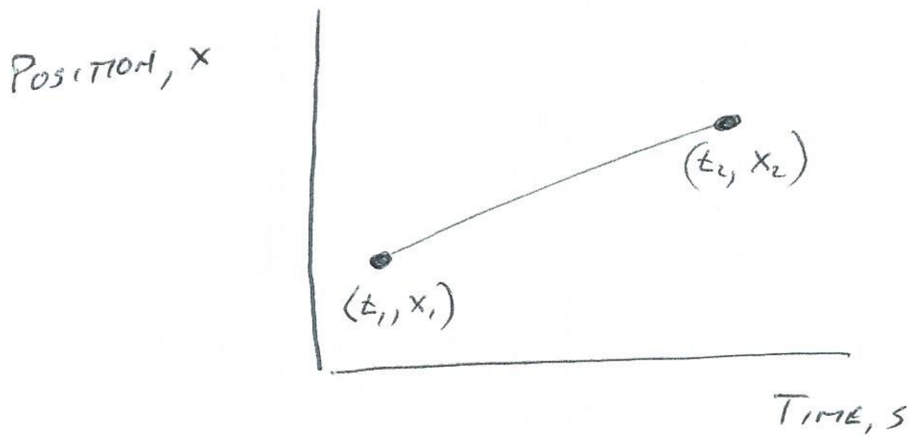
$$\Delta X_2 = \frac{0 - 20.8^2}{2(-6)}$$

$$\Delta X_2 = \underline{36.1\,m}$$

$$\Delta X_{TOTAL} = 12.5 + 36.1 = \underline{48.6\,m}$$

GRAPHICAL REPRESENTATION OF MOTION

POSITION-TIME GRAPH



OBJECT STARTS AT POSITION x_1 AT TIME t_1, MOVES TO x_2 AT t_2

$$\text{SLOPE} = \frac{\text{RISE}}{\text{RUN}} = \frac{x_2 - x_1}{t_2 - t_1}$$ ← THIS IS THE DEFINITION OF VELOCITY!

* ON A POSITION-TIME GRAPH (AN x-t GRAPH) THE SLOPE IS VELOCITY.

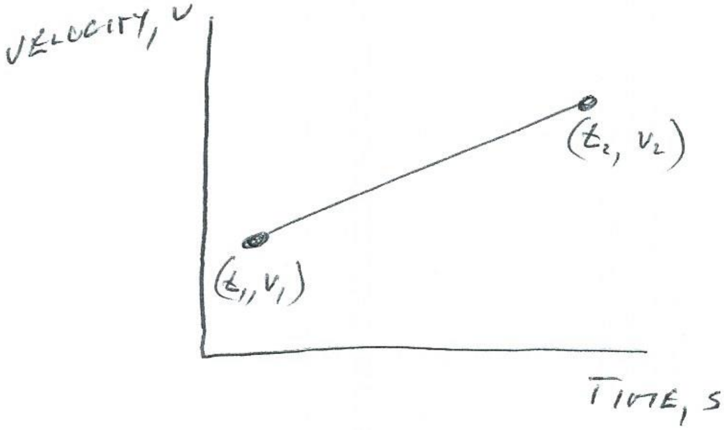
RECALL, $x = v\,t + x_0$

FROM MATH: $y = m\,x + b$

ON x-t GRAPHS, STRAIGHT LINES INDICATE CONSTANT VELOCITY.

CURVED LINES INDICATE A CHANGING VELOCITY (ACCELERATION).

VELOCITY-TIME GRAPH



$$\text{SLOPE} = \frac{\text{RISE}}{\text{RUN}} = \frac{v_2 - v_1}{t_2 - t_1}$$ ~ DEFINITION OF ACCELERATION!

* ON A V-t GRAPH, SLOPE IS ACCELERATION!

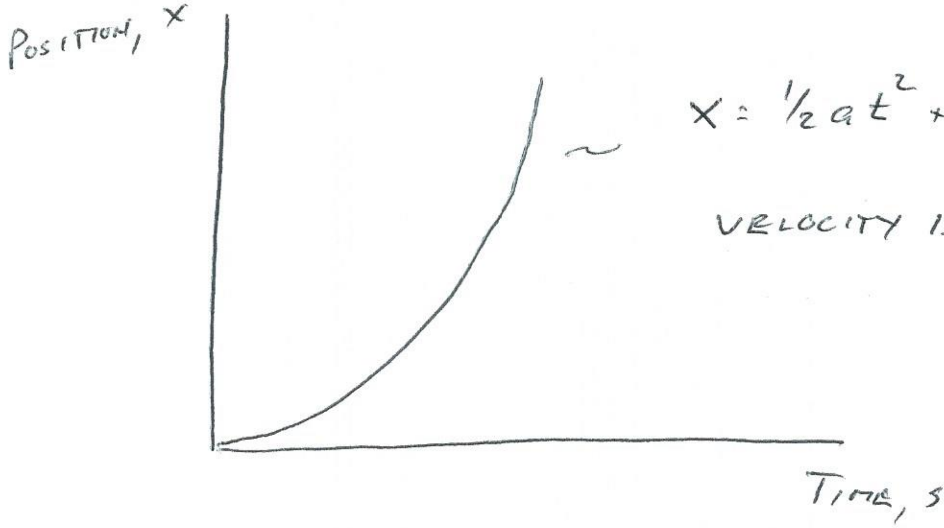
RECALL, $v = at + v_0$

$y = mx + b$

ON V-t GRAPHS, STRAIGHT LINES INDICATE CONSTANT ACCELERATION.

A CURVED LINE WOULD INDICATE A CHANGING ACCELERATION.

POSITION - TIME



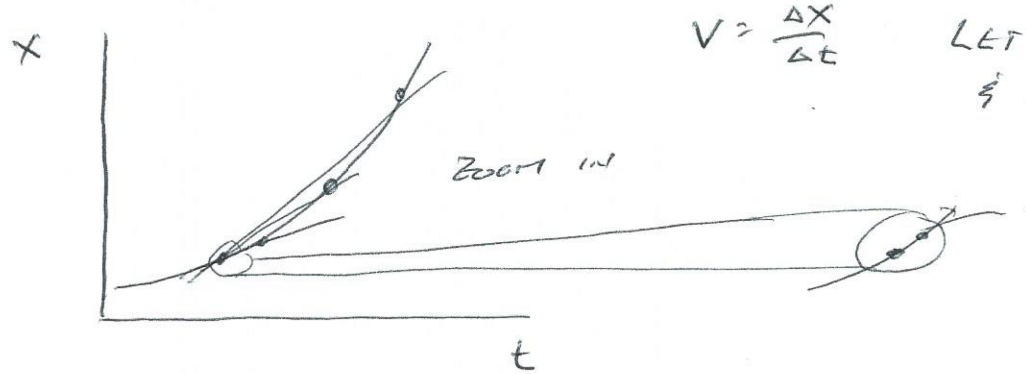
$$X = \tfrac{1}{2} a t^2 + V_0 t + X_0$$

VELOCITY IS CHANGING!

INSTANTANEOUS VELOCITY

$$V = \frac{\Delta X}{\Delta t}$$

LET Δt GET SMALLER & SMALLER



ALMOST A STRAIGHT LINE.

INSTANTANEOUS VELOCITY IS THE VELOCITY AT ONE POINT.

$$\boxed{V = \lim_{\Delta t \to 0} \frac{\Delta X}{\Delta t}}$$

1-D MOTION WITH CALCULUS

- WE WILL REVIEW THIS IN CLASS THE FIRST WEEK OF SCHOOL.

INSTANTANEOUS VELOCITY:

$$V = \lim_{\Delta t \to 0} \frac{\Delta X}{\Delta t}$$

AS THIS Δt BECOMES SMALLER & SMALLER, WE THEN SAY:

$$\boxed{V = \frac{dx}{dt}}$$ (REPLACE THE Δ's WITH d's)

IN CALCULUS, THIS IS CALLED A DERIVATIVE.

WE SAY VELOCITY IS THE (FIRST) DERIVATIVE OF POSITION.

LIKEWISE, IF V IS CHANGING,

$$a = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} \Rightarrow \boxed{a = \frac{dV}{dt}}$$

a IS THE (FIRST) DERIVATIVE OF VELOCITY.

BUT, $a = \frac{dV}{dt}$ & $V = \frac{dx}{dt}$

$$a = \frac{d\left(\frac{dx}{dt}\right)}{dt} \Rightarrow \boxed{a = \frac{d^2x}{dt^2}}$$

a IS THE SECOND DERIVATIVE OF POSITION.

WHAT DOES ALL THIS MEAN? IT IS QUITE SIMPLE IN THE END, ONCE YOU GET THE HANG OF IT. YOU WILL USE THE DERIVATIVE A <u>LOT</u> IN CALC & THERE ARE A NUMBER OF RULES TO FOLLOW.

HERE ARE ONES WE WILL GENERALLY USE, WITH EXAMPLES.

KEEP THIS IN MIND: A DERIVATIVE TELLS YOU HOW ONE VARIABLE IS CHANGING WITH RESPECT TO ANOTHER.

$V = \frac{dx}{dt}$ ~ HOW POSITION IS CHANGING OVER TIME.

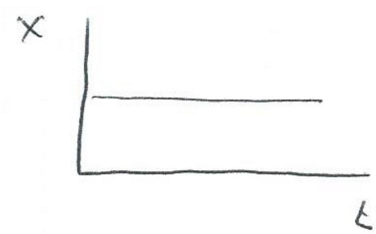
$a = \frac{dV}{dt}$ ~ HOW VELOCITY IS CHANGING OVER TIME.

<u>TWO RULES FOR DERIVATIVES</u>

(1) CONSTANT FUNCTION

IF X IS CONSTANT

$$\boxed{\frac{dx}{dt} = 0}$$



(2) POWER RULE

IF $x(t) = t^n$, THEN $\frac{dx}{dt} = n \cdot t^{n-1}$

<u>EX.</u>

$x(t) = t^3$ $\qquad$ $x(t) = 3t^2$ $\qquad$ $x(t) = 3t^2 + 5$

$\frac{dx}{dt} = 3t^2$ $\qquad$ $\frac{dx}{dt} = 6t$ $\qquad$ $\frac{dx}{dt} = 6t + 0$

EXAMPLES

① SUPPOSE AN OBJECT'S POSITION IS GIVEN BY

$$x = 3t^2 - 12t + 4$$

a) WHERE IS IT AT $t = 2s$?

$$x = 3(2)^2 - 12(2) + 4 \qquad \underline{x = -8m}$$

b) WHAT IS $v(t)$?

$$v = \frac{dx}{dt} = 6t - 12$$

c) WHAT IS ITS VELOCITY AT $t = 2s$?

$$v = 6(2) - 12 \qquad \underline{v = 0}$$

d) WHAT IS ITS $a(t)$?

$$a = \frac{dv}{dt} = \underline{6\ m/s^2} \qquad a \text{ IS CONSTANT!}$$

NOTICE HOW WE FOUND a BY USING CALC, IN d)
WE COULD HAVE DONE IT THIS WAY:

$$x = 3t^2 - 12t + 4$$

$$x = \tfrac{1}{2}at^2 + v_0 t + x_0$$

$$x_0 = 4m \qquad v_0 = -12\ m/s \qquad \tfrac{1}{2}a = 3$$

$$\underline{a = 6\ m/s^2}$$

Motion in y-direction

If we throw something straight up or down, or just let it fall, it moves in 1-D.

The difference now is we use y instead of x, and acceleration is gravity, g.

The average value of $g = -9.8\ m/s^2$ ← Very important!

Negative because it acts downward!

So now we have

$$\boxed{y = \tfrac{1}{2} g t^2 + V_0 t + y_0}$$

$$\boxed{V^2 = V_0^2 + 2g\Delta y}$$

Examples

① You drop a rock from the bridge over the Mississippi River. How high is the bridge?

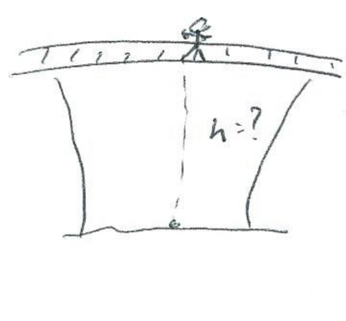
Suppose it takes 3.2 seconds to hit the water.

$h = \Delta y$

$V_0 = 0$

$t = 3.2s$

$g = -9.8\ m/s^2$



$$\Delta y = \tfrac{1}{2} g t^2 + \cancel{V_0 t}^{\,0}$$

$$\Delta y = +\tfrac{1}{2}(-9.8)(3.2)^2 + 0$$

$$\Delta y = -50.2\ m$$

Negative sign indicates it went downward!

EXAMPLE 2

SUPPOSE YOU THROW A BALL STRAIGHT UP WITH $V_0 = 25\,m/s$.

a) HOW HIGH WILL IT GO?

GIVEN:	FIND:	FORMULA
$V_0 = 25\,m/s$	Δy	$v^2 = v_0^2 + 2g\Delta y$
$g = -9.8\,m/s^2$		
$v = 0$ - AT HIGHEST POINT!		

$$v^2 = v_0^2 + 2g\Delta y$$

$$0 = 25^2 + 2(-9.8)\Delta y \quad \Rightarrow \quad 19.6\Delta y = 625$$

$$\underline{\Delta y = 31.6\,m}$$

b) HOW LONG IS IT IN THE AIR?

$$t_{TOT} = t_{UP} + t_{DOWN}$$

GIVEN:	FIND	FORMULA:
SAME AS ABOVE	t_{UP}	$V = gt + V_0$
		$t = \frac{v - V_0}{g}$

$$t_{UP} = \frac{0 - 25}{-9.8} \qquad \underline{t_{UP} = 2.55\,s}$$

SINCE GRAVITY IS THE ONLY THING THAT ACTS ON IT, $t_{UP} = t_{DOWN}$

SO $t_{TOTAL} = 2(2.55)$

$$\underline{t_{TOTAL} = 5.1\,s}$$

ALTERNATE SOLUTION:

USE $\Delta y = \frac{1}{2}gt^2 + V_0 t$ $\quad$ $\Delta y = 0$ SINCE IT STARTS & ENDS AT SAME POINT.

$$0 = \tfrac{1}{2}(-9.8)t^2 + 25t$$

$$t = 0 \text{ OR } t = 5.1\,s$$

CH. 3 - VECTORS

VECTORS - QUANTITIES THAT SHOW BOTH MAGNITUDE & DIRECTION.

MAGNITUDE ~ THE NUMERICAL VALUE

DIRECTION ~ INDICATED BY N, S, E, W (RARE)
OR AN ANGLE (COMMON)
OR TO THE RIGHT, LEFT
OR + OR -

SCALARS - QUANTITIES THAT SHOW MAGNITUDE ONLY.

EXAMPLES

VECTORS	SCALARS
DISPLACEMENT	DISTANCE
VELOCITY	SPEED
ACCELERATION	TIME
GRAVITY	ENERGY
FORCES	WORK
MOMENTUM	MASS
⋮	⋮

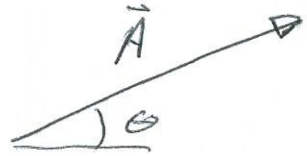
WE INDICATE VECTORS IN WRITING BY PUTTING AN ARROW (OR HALF-ARROW) OVER THE VARIABLE.

FOR EXAMPLE: $\vec{V}$ - VELOCITY

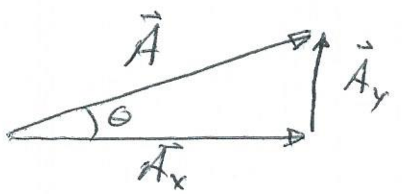
V - SPEED (NO ARROW ABOVE IT)

WHEN INDICATING VECTORS IN A DRAWING, THEY MUST BE SHOWN AS AN ARROW POINTING IN THE PROPER DIRECTION.

FOR EXAMPLE, FOR SOME RANDOM VECTOR $\vec{A}$ AT SOME ANGLE θ (RELATIVE TO THE +X AXIS), WE DRAW IT AS



IN THIS EXAMPLE $\vec{A}$ IS 'MOVING' OR 'POINTING' IN BOTH THE X-DIRECTION (TO THE RIGHT) & IN THE Y-DIRECTION (UP). WE SAY IT HAS COMPONENTS IN THE X & Y DIRECTION, $\vec{A}_x$ & $\vec{A}_y$



NOTICE, $\vec{A}_x \perp \vec{A}_y$

LOOKING ONLY AT MAGNITUDES

$$A_x^2 + A_y^2 = A^2$$ PYTHAGOREAN THEOREM!

ALSO, TRIGONOMETRY:

$$A_x = A\cos\theta$$

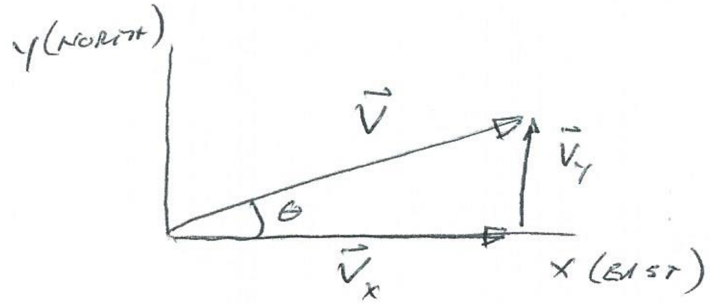
$$A_y = A\sin\theta$$

* WE WILL USE THESE RELATIONSHIPS OFTEN! *

EXAMPLE

SUPPOSE AN AIRPLANE IS FLYING AT $V = 300\ m/s$ AT $\theta = 25°$ NORTH OF EAST.

WHAT ARE ITS EASTERLY & NORTHERLY COMPONENTS?

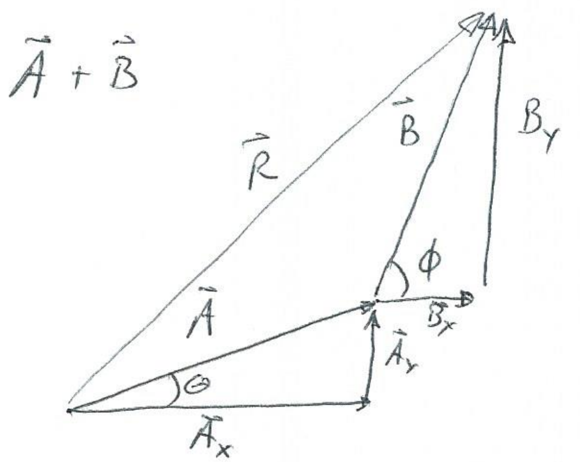


$V_x = 300 \cos 25° = 271.9\ m/s$ ~ EASTWARD SPEED

$V_y = 300 \sin 25° = 126.9\ m/s$ ~ NORTHWARD SPEED

ADDING VECTORS

YOU CAN ONLY ADD VECTORS OF THE SAME TYPE (VELOCITY + VELOCITY, FORCE + FORCE). YOU CANNOT ADD VECTORS OF DIFFERENT TYPES (VELOCITY + ACCELERATION).



DRAW TIP TO TAIL!

$\vec{A} + \vec{B}$ = RESULTANT, $\vec{R}$

$\vec{R}$ GOES FROM TAIL OF $\vec{A}$ TO TIP OF $\vec{B}$

- FIND COMPONENTS (x & y) OF BOTH $\vec{A}$ & $\vec{B}$ & PUT INTO TABLE ⟶
- ADD X COMPONENTS = R_x
- ADD Y COMPONENT = R_y
- PYTHAGORIZE TO FIND R

$$R = \sqrt{R_x^2 + R_y^2}$$

	X	Y
A	A_x	A_y
B	B_x	B_y
R	R_x	R_y

- FIND ANGLE, α, FOR R

$$\tan\alpha = \frac{R_y}{R_x} \qquad \alpha = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

EXAMPLE

AIRPLANE FLYING AT $V = 300\ m/s$, $\theta = 25^\circ$

WIND BLOWS AT $W = 80\ m/s$, $\phi = 60^\circ$

WHAT IS THE RESULTANT VELOCITY OF THE PLANE?

FROM PREVIOUS EXAMPLE

	X	Y
V	271.9	126.9
W	40	69.3
R	311.9	196.2

$$W_x = 80\cos 60 = 40\ m/s$$

$$W_y = 80\sin 60 = 69.3\ m/s$$

$$R = \sqrt{311.9^2 + 196.2^2}$$

$$\underline{R = 368.5\ m/s}$$

$$\alpha = \tan^{-1}\left(\frac{196.2}{311.9}\right) = 32.2^\circ$$

THE AIRPLANE WILL ACTUALLY TRAVEL AT <u>368.5 m/s AT 32.2°</u>!

'MULTIPLYING' VECTORS

WE'LL PRACTICE THE MATH FOR THIS NOW, THE APPLICATIONS COME LATER.

2 WAYS

DOT PRODUCT $\rightarrow$ RESULT IS A SCALAR

CROSS (VECTOR) PRODUCT $\rightarrow$ RESULT IS A VECTOR

FIRST, ANOTHER WAY TO REPRESENT VECTORS IS WITH 'UNIT VECTOR NOTATION'. A UNIT VECTOR HAS MAGNITUDE = 1, AND POINTS IN EITHER THE X, Y, OR Z AXIS.

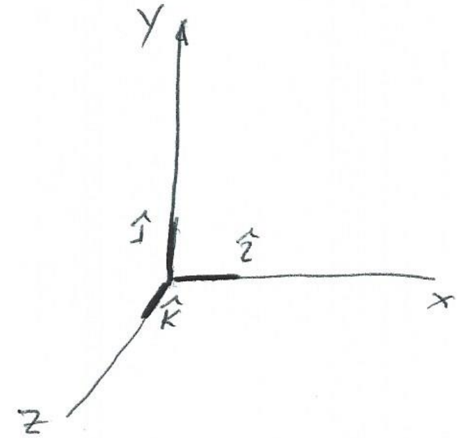
THE UNIT VECTOR IN X-DIR IS SHOWN AS $\hat{i}$

" Y-DIR " $\hat{j}$

" Z-DIR " $\hat{k}$

THE '^' SYMBOL INDICATES A UNIT VECTOR.

i, j, & k ARE CONVENTIONAL NOTATION.



SUPPOSE VECTOR $\vec{A}$ HAS COMPONENTS IN X, Y & Z DIR.

WE SHOW IT AS

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

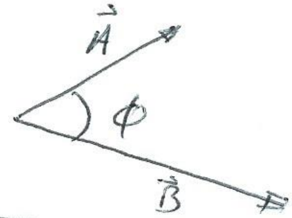
A_x, A_y, A_z ARE THE MAGNITUDES IN THOSE DIRECTIONS.

DOT PRODUCT

$$\vec{A} \cdot \vec{B} = A \cdot B \cos\phi$$

MAGNITUDES OF $\vec{A}$ & $\vec{B}$

ϕ IS ANGLE BETWEEN $\vec{A}$ & $\vec{B}$



MAGNITUDE OF $\vec{A}$: $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$$\vec{A} \cdot \vec{B} = A_x \cdot B_x + A_y \cdot B_y + A_z \cdot B_z$$

EXAMPLE

WHAT IS THE ANGLE BETWEEN $\vec{A}$ & $\vec{B}$ IF

$\vec{A} = -5\hat{i} + \hat{j}$ & $\vec{B} = 5\hat{i} + 4\hat{j}$

$$\vec{A} \cdot \vec{B} = -5(5) + 1(4) = -21$$

$$A = \sqrt{(-5)^2 + 1^2} = \sqrt{26}$$

$$B = \sqrt{5^4 + 4^2} = \sqrt{41}$$

$$-21 = \sqrt{26} \cdot \sqrt{41} \cos\phi$$

$$\cos\phi = \frac{-21}{\sqrt{26} \cdot \sqrt{41}} = -.643 \qquad \phi = \cos^{-1}(.643)$$

$$\boxed{\phi = 130^\circ}$$

CROSS PRODUCT - A LITTLE MORE CHALLENGING, BUT STILL FUN!

$\vec{A} \times \vec{B}$

GIVEN $\vec{A} = A_x \hat{\imath} + A_y \hat{\jmath} + A_z \hat{k}$

$\vec{B} = B_x \hat{\imath} + B_y \hat{\jmath} + B_z \hat{k}$

SET UP A MATRIX:

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \hat{\imath} - \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \hat{\jmath} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \hat{k}$$

$$= \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} \hat{\imath} - \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} \hat{\jmath} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} \hat{k}$$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{\imath} - (A_x B_z - A_z B_x)\hat{\jmath} + (A_x B_y - A_y B_x)\hat{k}$$

EXAMPLE

IF $\vec{A} = 3\hat{\imath} + 5\hat{\jmath}$ & $\hat{C} = 4\hat{\imath} + 3\hat{k}$

WHAT IS $\vec{A} \times \vec{C}$?

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 5 & 0 \\ 4 & 0 & 3 \end{vmatrix}$$

$$\vec{A} \times \vec{C} = (5\cdot 3 - 0\cdot 0)\hat{\imath} - (3\cdot 3 - 0\cdot 4)\hat{\jmath} + (3\cdot 0 - 5\cdot 4)\hat{k}$$

$$\boxed{\vec{A} \times \vec{C} = 15\hat{\imath} - 9\hat{\jmath} - 20\hat{k}}$$

Does $\vec{A} \times \vec{C} = \vec{C} \times \vec{A}$